Formulario di goniometria

Funzioni goniometriche di angoli particolari

Gradi	Radianti	Seno	Coseno	Tangente	Cotangente
$0°$	0	0	1	0	*non esiste*
$15°$	$\frac{\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$
$18°$	$\frac{\pi}{10}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\sqrt{\frac{5-2\sqrt{5}}{5}}$	$\sqrt{5+2\sqrt{5}}$
$22°30'$	$\frac{\pi}{8}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\sqrt{2}-1$	$\sqrt{2}+1$
$30°$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$36°$	$\frac{\pi}{5}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\sqrt{5-2\sqrt{5}}$	$\sqrt{\frac{5+2\sqrt{5}}{5}}$
$45°$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$54°$	$\frac{3}{10}\pi$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\sqrt{\frac{5+2\sqrt{5}}{5}}$	$\sqrt{5-2\sqrt{5}}$
$60°$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$67°30'$	$\frac{3}{8}\pi$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\sqrt{2}+1$	$\sqrt{2}-1$
$72°$	$\frac{2}{5}\pi$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\sqrt{5+2\sqrt{5}}$	$\sqrt{\frac{5-2\sqrt{5}}{5}}$
$75°$	$\frac{5}{12}\pi$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$	$2-\sqrt{3}$
$90°$	$\frac{\pi}{2}$	1	0	*non esiste*	0
$180°$	π	0	-1	0	*non esiste*
$270°$	$\frac{3}{2}\pi$	-1	0	*non esiste*	0
$360°$	2π	0	1	0	*non esiste*

Relazioni fondamentali tra le funzioni goniometriche di uno stesso angolo

$$\sin^2\alpha + \cos^2\alpha = 1 \qquad \tan\alpha = \frac{\sin\alpha}{\cos\alpha} \qquad \cot\alpha = \frac{\cos\alpha}{\sin\alpha} \qquad \operatorname{cosec}\alpha = \frac{1}{\sin\alpha} \qquad \sec\alpha = \frac{1}{\cos\alpha}$$

Funzioni goniometriche di angoli associati

$\sin(-\alpha) = -\sin\alpha$	$\cos(-\alpha) = \cos\alpha$	$\tan(-\alpha) = -\tan\alpha$	$\cot(-\alpha) = -\cot\alpha$
$\sin(2\pi-\alpha) = -\sin\alpha$	$\cos(2\pi-\alpha) = \cos\alpha$	$\tan(2\pi-\alpha) = -\tan\alpha$	$\cot(2\pi-\alpha) = -\cot\alpha$
$\sin(\pi-\alpha) = \sin\alpha$	$\cos(\pi-\alpha) = -\cos\alpha$	$\tan(\pi-\alpha) = -\tan\alpha$	$\cot(\pi-\alpha) = -\cot\alpha$
$\sin(\pi+\alpha) = -\sin\alpha$	$\cos(\pi+\alpha) = -\cos\alpha$	$\tan(\pi+\alpha) = \tan\alpha$	$\cot(\pi+\alpha) = \cot\alpha$
$\sin\left(\frac{\pi}{2}-\alpha\right) = \cos\alpha$	$\cos\left(\frac{\pi}{2}-\alpha\right) = \sin\alpha$	$\tan\left(\frac{\pi}{2}-\alpha\right) = \cot\alpha$	$\cot\left(\frac{\pi}{2}-\alpha\right) = \tan\alpha$
$\sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha$	$\cos\left(\frac{\pi}{2}+\alpha\right) = -\sin\alpha$	$\tan\left(\frac{\pi}{2}+\alpha\right) = -\cot\alpha$	$\cot\left(\frac{\pi}{2}+\alpha\right) = -\tan\alpha$

Formule di addizione e sottrazione

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha \qquad \sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\begin{cases} \tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} \\ \alpha+\beta, \alpha, \beta \neq \dfrac{\pi}{2} + k\pi \end{cases} \qquad \begin{cases} \tan(\alpha-\beta) = \dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta} \\ \alpha-\beta, \alpha, \beta \neq \dfrac{\pi}{2} + k\pi \end{cases}$$

Formule di duplicazione

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$$

$$\begin{cases} \tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha} \\ \alpha \neq \dfrac{\pi}{4} + k\dfrac{\pi}{2} \wedge \alpha \neq \dfrac{\pi}{2} + k\pi \end{cases}$$

Formule di bisezione

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}} \qquad \sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}} \qquad \tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} \quad (\alpha \neq \pi + 2k\pi)$$

$$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha} \quad \text{con } \alpha \neq \pi + 2k\pi$$

$$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} \quad \text{con } \alpha \neq k\pi$$

Formule parametriche

$$\sin\alpha = \frac{2t}{1+t^2} \qquad \cos\alpha = \frac{1-t^2}{1+t^2} \qquad \left(t = \tan\frac{\alpha}{2}, \quad \alpha \neq \pi + 2k\pi\right)$$

Formule di prostaferesi

$$\sin p + \sin q = 2\sin\frac{p+q}{2}\cos\frac{p-q}{2} \qquad \cos p + \cos q = 2\cos\frac{p+q}{2}\cos\frac{p-q}{2}$$

$$\sin p - \sin q = 2\cos\frac{p+q}{2}\sin\frac{p-q}{2} \qquad \cos p - \cos q = -2\sin\frac{p+q}{2}\sin\frac{p-q}{2}$$

Formule di Werner

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$